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FUNCTIONAL OF VOLTERRA FOR REPRESENTATION OF AIRCRAFT NON-LINEAR DYNAMICS

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Abstract. *This work presents an approach for the systematic identification of the non linear dynamics of aircraft through a functional of Volterra. It is demonstrated that the functional of Volterra can be used to represent the continuous time dynamic answer of any non linear invariant system. Besides, the functional of Volterra also propitiates the description of the system in terms of bilinear systems. This representation offers larger possibilities to project control of non linear systems. The rigidity of the mathematical formulation of the functional of Volterra has been motivating several functional representations of different forms to extend the applicability of the models of dynamic systems. Recently, with the progresses in the theory of computational neural networks, a peculiar architecture of neural networks was developed in a way to be equivalent to a discreet time series of Volterra. This methodology facilitates the determination of the kernel of Volterra for any wanted order. The neural network approach, to achieve a Volterra series, is applied for the case of an aircraft non-linear longitudinal dynamics. Results have shown that the approach performs well, provides suitable approximation of the non-linear behaviour of aircraft longitudinal dynamics and further applications in aircraft non-linear analysis and control design.*

Key-words: *Volterra functional, Identification, Aircraft non linear dynamics*

1. INTRODUCTION

An aircraft is a complex aggregate of elastic bodies subjected to a complicated system of external non-linear loads and inertial effects. A complete model that can be used to calculate all these effects is still not practical for industrial applications. Much of the aircraft modelling schemes serving either research or industry, present many simplifications that limit the design and analysis of aircraft non-linear behaviour at high angles of attack or at transonic regimes.

An alternative to overcome such modelling problems is to use techniques from non-linear systems identification theory. Aircraft dynamics identification has reached today a high level of development, mainly due to advanced measurement and data processing techniques (Hamel & Jategaonkar, 1996). Approaches for aircraft dynamics identification have commonly treated the aerodynamic loading using linear parameter methods (Klein, 1989). While the use of non-linear identification strategies for general dynamic systems has been of great interest, only a few preliminary studies of non-linear aircraft flight dynamics identification have appeared in the literature (Hamel & Jategaonkar, 1996).

Mathematical techniques in non-linear identification, such as the Volterra functional theory (Volterra, 1959) of non-linear systems, furnishes a rigorous formulation. So, supported by this mathematical formality, Tobak and Schiff (1981) have proposed the indicial response functional to compose aerodynamic response histories.

Volterra (1959) has also shown that expansions of the definition of the Taylor series for a function can be generalised to functionals. The resulting functional series is the so-called, *Volterra series*. Volterra (1959) has also proved that any continuous, causal, time-invariant non-linear system can be modelled as an infinite sum of multi-dimensional convolution integrals of increasing order; that is, the Volterra series itself. The great drawback of Volterra series, however, is the calculation of its *kernels* (Billings, 1980, Schetzen, 1980).

Related to expansions of the Volterra type of functional series, other methodologies have been developed (Billings, 1980). The Wiener methods (Schetzen, 1980, 1981) provide potential identification schemes for non-linear dynamic systems but with the excessive number of coefficients required to identify the functional series, this methods are impractical and difficult to apply.

Other techniques for non-linear dynamic systems identification are based on block-oriented models (Billings & Fakhouki, 1979, Billings, 1980, Hunter & Korenberg, 1986). These approaches represent systems as cascade structures of combinations of linear dynamic and non-linear static subsystems. The Hammerstein model (Billings & Fakhouki, 1979, Hunter & Korenberg, 1986) is a block-oriented representation of non-linear systems. Similarly, system models that consist of a cascade of a linear dynamic subsystems, a static non-linearity, and another linear dynamic subsystem (Korenberg & Hunter, 1986), furnish another possible approach in non-linear identification by combining the ideas from Wiener and Hammerstein cascade models.

These techniques have been developed strictly for random processes and an important drawback associated to the aforementioned methods is the difficulty in determining the large number of identification parameters.

Recently, studies using *artificial neural networks* (Haykin, 1994) for identification have shown great potential for non-linear systems modelling, either on its own or assisting other types of approaches. The application of neural networks to identify Volterra functional series has been proposed by Wray and Green (1994). It has been proved that particular network architecture can be equivalent to a Volterra series representation of a dynamic system. Moreover, the kernels of any order can also be extracted from the network parameters.

So, the aim of this paper is to present an investigation on a systematic identification approach for Volterra functional series representations of aircraft non-linear dynamics. The identification procedure is carried out by means of a supervised network training scheme using standard back-propagation algorithm (Haykin, 1994). The approach is used to identify an aircraft non-linear longitudinal dynamics representation from data obtained in simulations of the aircraft longitudinal equations of motion accounting for non-linear coupling effects and linearised aerodynamic reactions. The training pattern comprises motion-induced horizontal velocity response history to variations of the aircraft elevator angle.

2. VOLTERRA SERIES

Volterra series (Volterra, 1959) are functional forms developed as a generalisation of the Taylor series expansion for a function. The Volterra functional series approach is based on that an exact description of a continuous non-linear, physically realisable, time-invariant system is provided by an infinite series of multi-dimensional convolution integrals of increasing order expressed as:

$$
\begin{aligned}
y(t) = h_0 + & \int_{-\infty}^{\infty} h_1(\tau_1)\,u(t-\tau_1)\,d\tau_1 + \\
& \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)\,u(t-\tau_1)\,u(t-\tau_2)\,d\tau_1 d\tau_2 + \ldots + \\
& \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_n(\tau_1,\ldots,\tau_n)\,u(t-\tau_1)\ldots u(t-\tau_n)\,d\tau_1\ldots d\tau_n + \ldots
\end{aligned}
\tag{1}
$$

where $y(t)$ is the system response, $u(t)$ is the input to the system, and h_n is the n^{th}-order Volterra kernel.

The Volterra kernels are functions of the variables τ_i and each one represents a measurement of the systems non-linearity. The zeroth-order Volterra kernel, h_0, is a constant equal to the zero-input response of the system. The first-order Volterra kernel represents the linear response of the system to a unit impulse input, while the higher-order kernels are the non-linear responses of the system to multiple unit impulse inputs.

For causal systems, if any of the $\tau_1,...,\tau_n$ is less then zero, then the kernel $h_n(\tau_1,...,\tau_n)$ is zero and the lower limits of the integrals in Eq. (1) can be set equal to zero. Moreover, with no loss of generality, it is possible to assure that each kernel $h_n(\tau_1,...,\tau_n)$ in Eq. (1) is symmetric with respect to any permutation of $\tau_1,...,\tau_n$.

The Volterra series for causal, finite memory, T, time-invariant and discrete time systems is:

$$
\begin{aligned}
y(t) \cong h_0 + & \sum_{\tau_1=0}^{T} h_1(\tau_1)\,u(t-\tau_1) + \\
& \sum_{\tau_1=0}^{T}\sum_{\tau_2=0}^{T} h_2(\tau_1,\tau_2)\,u(t-\tau_1)\,u(t-\tau_2) + \ldots + \\
& \sum_{\tau_1=0}^{T}\cdots\sum_{\tau_n=0}^{T} h_n(\tau_1,\ldots,\tau_n)\,u(t-\tau_1)\ldots u(t-\tau_n) + \ldots
\end{aligned}
\tag{2}
$$

Various methods to assess the Volterra kernels have been developed, as it can be seen in Billings (1980) or in Schetzen (1980). Some of the discussed and reviewed approaches in the aforementioned research works are: *(i)* kernel estimation of a finite-order system using multiple pulse inputs and repeated experiments; *(ii)* kernel approximation by an expansion of orthogonal functions, with coefficients determined by gradient-type algorithms and pattern recognition methods; *(iii)* discrete Volterra kernels determination in terms of multi-dimensional *z*-transforms using high-order correlation functions and coloured Gaussian inputs. In contrast with the generality features of Volterra functional series and their approximation properties, one difficulty is that the required kernel order may need to be very large to achieve a specified accuracy over the given set of inputs to outputs.

An alternative approach for systematic determination of Volterra kernels of any order has been proposed by Wray and Green (1994) by employing the theory of artificial neural networks. They have achieved particular network architecture that is equivalent to a Volterra series. The approach provides a way to extract the kernels of all dimensions of a non-linear system. The developments of the approach are outlined in the next section.

3. NEURAL NETWORKS

Based on neuro-biology concepts, neural networks are sets of interconnected processing units, called *neurons*. A typical representation for a generic neuron j, where $x_1,x_2,...,x_p$ are the stimulus signals, $w_{j1},w_{j2},...,w_{jp}$, are the synaptic weights, θ_j is a bias value, v_j is the activation potential, o_j is the neuron output signal, and $\varphi(.)$ is the activation function, is:

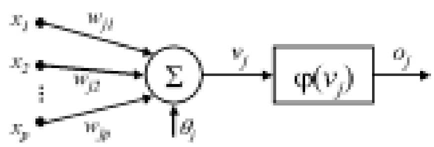

Figure 1 – Typical neuron representation.

where:

$$v_j = \theta_j + \sum_{i=1}^{p} w_{ji} x_i \qquad \text{and} \qquad o_j = \varphi(v_j)$$

Typical neural networks have the following architecture: (1) *input layer* – where the input stimulus is presented to the network; (2) *hidden layers* – internal layers of a network, and (3) *output layer* – the last layer of the network, where the outputs are given. Such typical network architecture is commonly referred as a *multi-layer neural network*.

To perform a desired task, the synaptic weights of a network must be initialised and modified by a *training* algorithm.

3.1 Neural network equivalent to volterra series

Wray and Green (1994) have presented a methodology of extracting the n^{th}-order Volterra kernels using neural networks. For a SISO dynamic system, the neural network equivalent to a Volterra series is illustrated in Fig. 2, where N is the number of time-delays of the input series, M is the number of hidden neurons, $u(t-j)$, for $j=0,...,N$, is the input u at time-delay j, w_{ji} is the synaptic weight in the connection between input u at time-delay j and the hidden neuron i, c_i is the synaptic weight in the connection between the hidden neuron i and the output neuron, and $y(t)$ is the network output at current time. The hidden neurons and the output unit present sigmoidal and linear activation functions respectively.

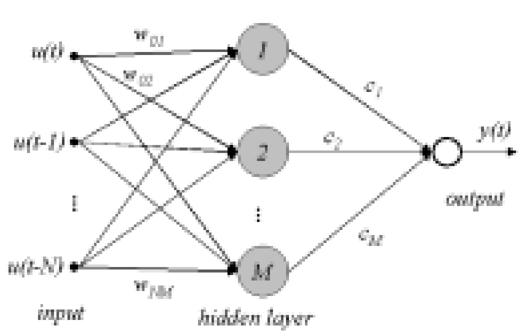

Figure 2 – Network architecture equivalent to a finite memory, discrete Volterra series.

The Wray and Green (1994) approach is based on that the activation of the hidden neurons can be approximated by a n^{th}-order polynomial function. So, the network output can be given as:

$$y(t)=\sum_{j=1}^{M} c_j\left(\sum_{k=0}^{n} a_{kj} v_j^k\right) \tag{3}$$

where a_{kj} are the polynomial coefficients and,

$$v_j=\theta_j+\sum_{i=0}^{N} w_{ji} u(t-i) \tag{4}$$

By expanding Eq.s (3) and (4) and grouping appropriate terms, the network (*cf.* Fig. 2) is equivalent to a finite memory discrete Volterra series (Wray & Green, 1994). Moreover, the kernels up to the n^{th}-order in Eq. (2), can be extracted according to the following expression:

$$h_n(\tau_1,\tau_2,\ldots,\tau_n)=\sum_{j=1}^{M} c_j\, a_{nj}\; w_{\tau_1 j}\; w_{\tau_2 j}\cdots w_{\tau_n j} \tag{5}$$

Here, the activation functions of the network hidden neurons shown in Fig. 2, are adopted as ($\beta = 0{,}8$) and:

$$\varphi(v)=\left(\frac{2}{1+e^{-2\beta v}}\right)-1 \tag{6}$$

Using the *least squares method* (Borse, 1991), the n^{th}-order polynomial approximation of the activation function given in Eq. (6) can be obtained.

3.2 Non-linear identification via volterra series

To illustrate the approach, a Volterra series representation of an aircraft longitudinal dynamics using a neural network is identified. The formulation used to simulate the aircraft non-linear dynamics is obtained from the work of Etkin and Reid (1996). It is assumed decoupled longitudinal motion and linearised aerodynamic reactions. The non-linear effects in the simulation are those originated by the coupling of pitch angular velocity with both horizontal and vertical scalar velocities. The resulting small disturbances equations are:

$$\Delta\dot{u}=\frac{X_u\Delta u+X_w w+X_q q+X_{\delta_e}\delta_e}{m}-g\cos\theta_0\theta-qw \tag{7a}$$

$$\dot{w}=\frac{Z_u\Delta u+Z_w w+Z_q q+Z_{\delta_e}\delta_e-mg\sin\theta_0\theta+mq\Delta u}{m-Z_{\dot{w}}} \tag{7b}$$

$$\dot{q}=\frac{M_u\Delta u+M_w w+M_{\dot{w}}\dot{w}+M_q q+M_{\delta_e}\delta_e}{m} \tag{7c}$$

$$\dot{\theta}=q \tag{7d}$$

where, u, w are the horizontal and vertical scalar velocities of the aircraft centre of gravity, q is the pitch angular velocity, θ is the pitch angle, θ_0 is the reference pitch angle, δ_e is the elevator angle, X_i, Z_i, M_i, are the derivatives of the resultant aerodynamic reactions with respect to the i^{th} variable, m is the aircraft mass, g is the gravity acceleration, I_y is the inertia moment, dot symbols represent $d(\ )/dt$, and Δ means small perturbations.

The data is obtained from longitudinal dynamics simulation (Etkin & Reid, 1996) of a Boeing 747-100 cruising horizontally at an altitude of approximately 12.000 *metres* and at a Mach number of 0,8. The velocity response $u(t)$, due to variations in the elevator angle $\delta_e(t)$ is considered in the identification process.

This identification process, shown in Fig. 3, commences with an initialisation of the synaptic weights using random uniform distribution (-1,0 to 1,0).

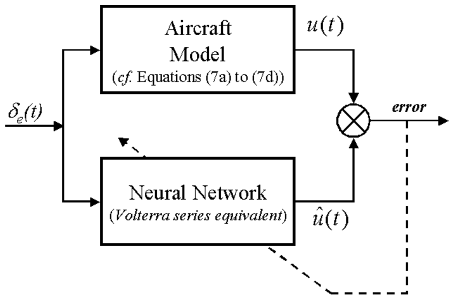

Figure 3 – Identification scheme.

Back-propagation algorithm (Haykin, 1994) is used to adapt the weights. To speed up the process, adaptive learning rate and momentum are also incorporated to the algorithm (Haykin, 1994). Normally, in non-linear systems identification schemes (Billings, 1980), input signals must present specific statistical properties so that the main dynamic features of the systems can be explored during the process. Indeed, many techniques (Billings, 1980) have been developed strictly for white Gaussian input forms. In the case of aircraft identification, such approaches are questionable, but although the employment of random input forms to aircraft identification schemes may not be practical, here these types of inputs are chosen because of their mathematical value. For the identification process, band-limited white noise is assumed to compose the input motion. The maximum absolute amplitude for the elevator angle is limited to a range of 1,5° to 2,0°. The training pattern has a sample interval of 1,0 *second*.

In order to assist the identification process, the input/output time-series pattern for the network training is subdivided into two parts. One part is used for the training (*training set*) while the other one is used to perform measurements to the resulting network output (*test set*). Time windows depending on the assumed number of time-delays, *N*, are presented to the network and the resulting error measure is used in the back-propagation algorithm. To compose the input, training pattern noise power value of 0,0005 and sample time of 2π are adopted. The neural network training parameters are: 10 time delays (*N*) in the input series, 50 hidden neurons (*M*), starting learning rate of 0,01; and starting momentum constant of 0,85. The training has been carried out in 10.000 epochs.

Figure 4 presents a comparison between the aircraft non-linear response obtained by simulations of the Eq.s (7a) to (7d) and the respective Volterra series representation after completion of the neural network training process. The resulting neural network can also be used to extract the kernels in such a way that the first- and second-order Volterra kernel approximations of the identified aircraft non-linear dynamics model can be calculated. Figures 5 and 6 depict the two kernels, respectively. To test the robustness of the identified model, a set of arbitrary inputs are applied to the neural network and Fig.s 7 to 11 show the results of simulation.

4. DISCUSSIONS

The ability of the neural network model to capture the essential features of the non-linear aircraft longitudinal dynamics can be observed in emulations for a broad range of motion-induced history used during the training process. In contrast to the time demanded by the training process to identify the model, the final network evaluations are fast enough to allow real-time predictions of non-linear dynamic system responses, justifying applications in analysis and control design. Moreover, the Volterra series approach allows subsequent bilinear representation.

For the identified neural network model the non-linear behaviour of the horizontal scalar velocity response is adequately captured. Few discrepancies can be observed. The predictive capabilities of the identified network model are satisfactory. Examining the first two Volterra kernel approximations, one can observe that the first-order kernel (*cf*. Fig. 5) represents the linear component of the Volterra series associated to the unit impulse response to the system. The second-order kernel (*cf*. Fig. 6) gives an idea of how the non-linear effects are distributed.

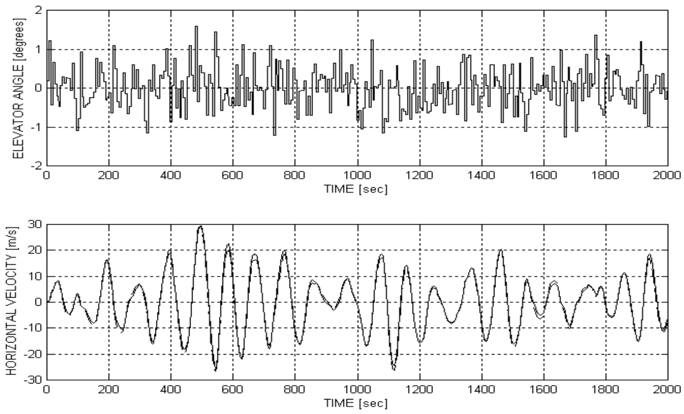

Figure 4 – Horizontal scalar velocity response due to elevator angle motion - training results (solid: aircraft model simulation; dashed: neural network output).

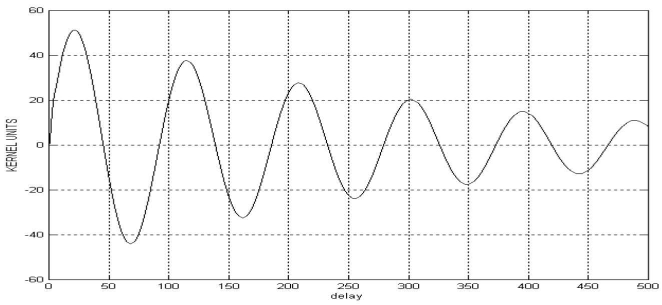

Figure 5 – First-order kernel of the Volterra series representation of the horizontal scalar velocity response to variations of the elevator angle.

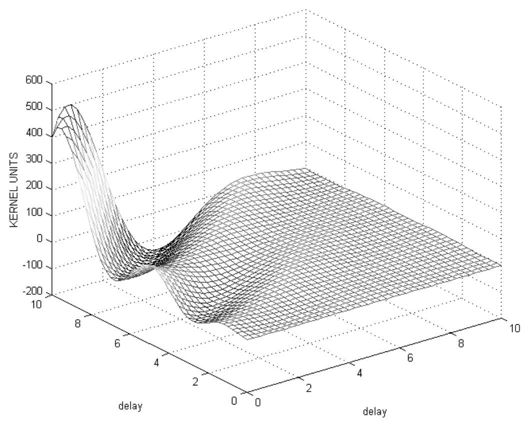

Figure 6 – Second-order kernel of the Volterra series representation of the horizontal scalar velocity response to variations of the elevator angle.

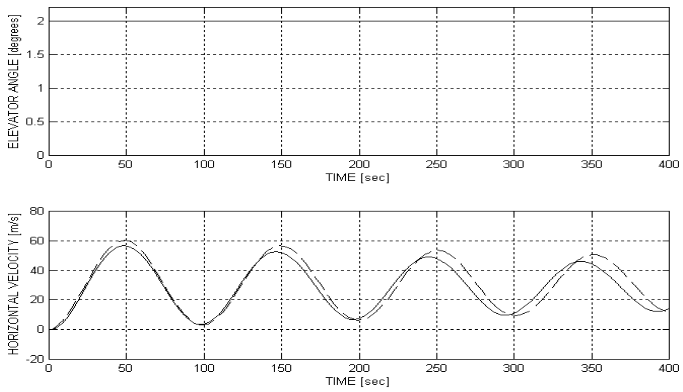

Figure 7 – Horizontal velocity response to a 2° amplitude elevator angle step input (solid: aircraft model simulation; dashed: neural network output).

When tested with a 2° amplitude step-input elevator motion (*cf.* Fig. 7) which leads to a typical phugoid response, the identified model reveals reasonable approximation of the horizontal velocity response. The delay encountered between the desired and predicted responses can be related to a loss of frequency description in the training pattern, or the need for more time-delays (*N*) in the composition of the inputs to the neural network.

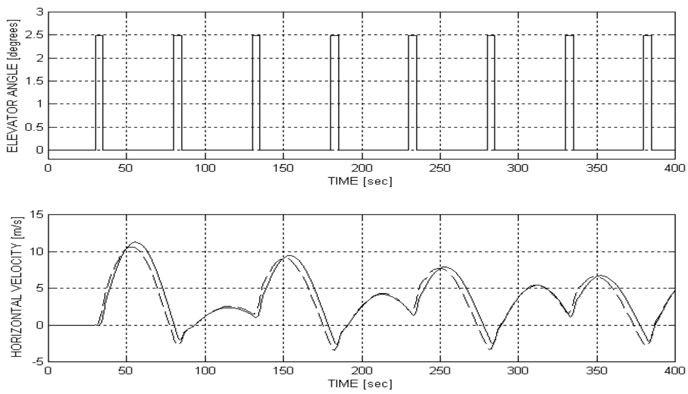

Figure 8 - Horizontal velocity response to a 2,5° amplitude, 5*s* at max. amplitude, 50*s* period, elevator angle squared pulses input (solid: model simulation; dashed: neural network output).

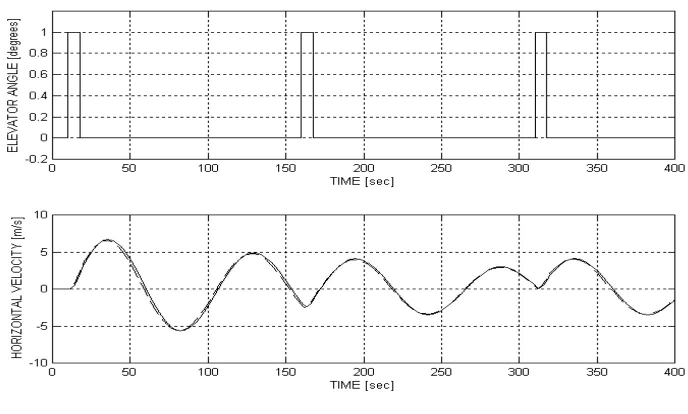

Figure 9 - Horizontal velocity response to a 1° amplitude, 10*s* at max amplitude, 150*s* period, elevator angle squared pulses (solid: model simulation; dashed: neural network output).

Figures 8 and 9 present generalisation tests for squared pulse types of elevator motion. The case in Fig. 8 corresponds to the velocity response *u(t)* to 2,5° amplitude squared pulses, starting at 30 *s*, 5 *s* duration in maximum amplitude, and period of 50 *s*. The identified model reveals a time lead-lag with respect to the desired response. In the case of Fig. 9, the identified network model satisfactorily predicted all the features of *u(t)* within the training limits.

The behaviour of the identified model is also tested for sinusoidal inputs of the elevator angle with different amplitudes and frequencies (Fig.s 10 and 11). Despite some differences, the identified network model maintains the main features of *u(t)* history. It can be observed that frequency and amplitude of the input motion history influence the prediction capabilities of the identified model. As far as amplitude is concerned, the same kinds of discrepancies observed in the previous tests also appear on the sinusoidal-type of induced responses.

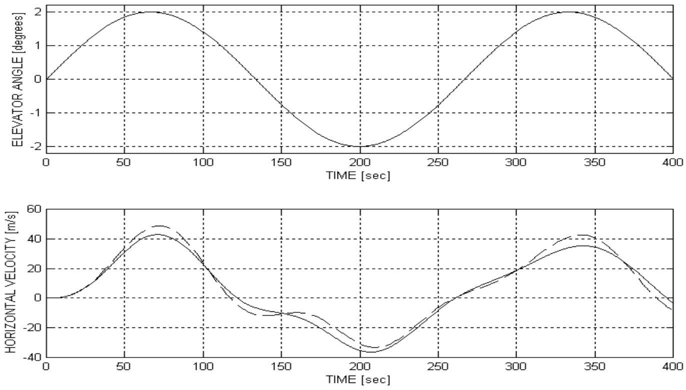

Figure 10 – Horizontal velocity response to a 2° amplitude, 0,00375 *Hz* frequency, elevator angle sinusoidal input (solid: aircraft model simulation; dashed: neural network output).

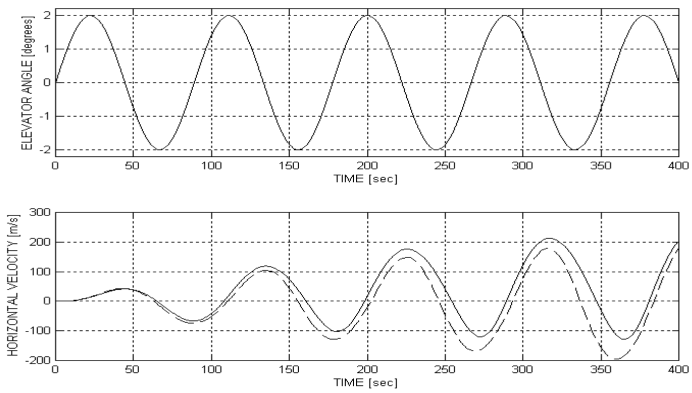

Figure 11 – Horizontal velocity response to a 2° amplitude, 0,015 *Hz* frequency, elevator angle sinusoidal input (solid: aircraft model simulation; dashed: neural network output).

The main features of the motion-induced responses have been all captured. Some large errors can be observed in cases with certain extreme frequency and amplitude values (*cf*. Fig. 11). Particularly, in Fig. 11 the horizontal scalar velocity unstable response reveals that in these circumstances, the model comprehensively fails to predict the combined frequency and amplitude features of the system response. Interestingly, the identified network model has been capable of predicting the instability, from what can be inferred that the resulting model provides reasonable representation of the system non-linear dynamic characteristics.

5. CONCLUSIONS

Volterra functional series, obtained by an equivalent neural network model, provides a suitable representation of an aircraft non-linear longitudinal dynamics. Supported by the rigorous concepts underlying the Volterra functional series approach to non-linear systems identification, the successful application of neural networks to determine an equivalent representation ensures both reliability and accurate mathematical description for the resulting model. Comparing this approach to other non-linear identification methods, the neural network technique has provided a much more powerful tool for a systematic assessment of equivalent Volterra functional series of non-linear dynamic systems representation.

The principal advantages of the approach are associated to the premise that an exact description for a continuous non-linear, causal, time-invariant system can be furnished by a Volterra functional series.

Generalisation tests have shown good predictive capabilities of the identified model within the training limits. The identified model has also presented adequate performance in a test case where the input motion induced to unstable condition of the aircraft response. For the aims of this investigation these results can be considered satisfactory.

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